Revisiting Productivity Growth Accounting Decompositions*

Filippo Massari†
Fairfield University

August 5, 2024

Abstract

Productivity growth accounting decompositions are sensitive to the definition of aggregate productivity, which usually takes the form of a weighted geometric or arithmetic average of firms' productivities. This paper shows explicitly why the two differ. It further proposes a modification to a popular decomposition highlighting the contribution of the component responsible for this discrepancy, namely the covariance of productivity growth rates and levels. The argument favoring this modified decomposition is that the novel component offers relevant information for applied theoretical purposes, thus increasing the usefulness of aggregate productivity decompositions. Finally, the paper argues that standard theoretical assumptions support (i) a proposed separation between aggregate and average productivity, and (ii) the adoption of the arithmetic average definition as opposed to the more popular geometric average.

JEL: D24, E24, O47.

Keywords: Productivity Growth Accounting, Aggregate Productivity, Average Productivity, Growth Decompositions.

*Acknowledgement: I thank Pietro Peretto, Giuseppe Fiori, James Nason, and Domenico Ferraro for comments on a much earlier version of this paper. All errors are my own.

†Department of Economics, Dolan School of Business, Fairfield University, 1073 North Benson Road, Fairfield, CT 06824. E-mail: fmassari@fairfield.edu. Website: http://filippomassari.com/.

1 Introduction

What explains changes in aggregate productivity? Economists and practitioners employ productivity growth accounting techniques to decompose aggregate productivity as a first diagnostic tool to guide further analysis. These decompositions illustrate how much of aggregate productivity growth is associated with improved firm-level productivity or reallocation of resources across incumbents, entering, and exiting firms.[1] Furthermore, these decompositions allow determining moments from the data helpful in calibrating or selecting the most appropriate models (Bartelsman et al., 2013).

The most widely used decompositions are Foster et al. (2001) and the dynamic Olley-Pakes presented in Melitz and Polanec (2015). They adopt an arbitrary definition of average productivity as the weighted geometric average of firms' productivity levels (alternatively expressed as the arithmetic average of firms' productivity log-levels), with market or labor shares as weights.[2] They then decompose average productivity growth into the various components. However, the definition of average productivity may be questioned due to the sensitivity of results under alternative definitions. For example, the Melitz and Polanec (2015) appendix provides different results for their exercise when defining average productivity as the arithmetic average of productivity levels. However, it does not analyze the nature of these differences because the papers' conclusions are unaltered.

In this paper, I argue that the appropriate definition of average productivity is the weighted arithmetic average based on standard economic theory. Furthermore, I propose a useful modification to the dynamic Olley-Pakes decomposition.

After explicitly showing that both the growth rate of average productivity and its decomposition are sensitive to its definition, I isolate the component responsible for this discrepancy. This component is the correlation between firms' productivity growth rates and levels. If the two are uncorrelated, the two definitions yield exactly the same results. In case of a negative

[1]Some recent and valuable examples of how these decompositions are employed include an assessment of productivity dynamics during the Covid pandemic (Bloom et al., 2023), making sense of sluggish Italian productivity growth (Bugamelli et al., 2018), understanding the nature of Chinese productivity growth Gao et al. (2023).

[2]Although in this stream of literature the term "aggregate" is used to refer to average productivity, for the remainder of the paper I will use the term "average", in line with the criticism levied, among others, by Osotimehin (2019) and Baqaee and Farhi (2020). In the paper, I show that aggregate productivity can be decomposed into average productivity and an aggregation term, thus justifying formally my nomenclature.

(positive) correlation, instead, I prove that the geometric average definition yields a within component for the dynamic Olley-Pakes decomposition that is always higher (lower), after a correction for entry and exit.[3] Furthermore, under a time-invariant firm-size distribution, the geometric average always grows faster (slower) than the arithmetic average.

The object of this paper is relevant for two reasons. First, an accurate understanding of productivity growth dynamics requires careful methodological considerations if different methodologies yield different results. Indeed, there are good reasons to suspect that these differences are present. The absence of any correlation between firms' employment or sales growth rates and their levels is known in the literature as Gibrat Law, which empirical studies have repeatedly rejected finding a negative correlation between the two and a firm-size distribution that is consistent with a negatively scale dependent firm growth process (Sutton, 1997; Caves, 1998; Audretsch et al., 2004; Rossi-Hansberg and Wright, 2007). Meanwhile, there is evidence of a positive correlation between firms' size and their productivity level (Bartelsman et al., 2013), which is also predicted by most theoretical setups. These two pieces of evidence combined suggest that firms' productivity growth and levels are negatively correlated, making the theme of this paper a topic of interest, although the size of this discrepancy remains to be seen.

Second, the theoretical growth literature includes models that align with Gibrat Law (Klette and Kortum, 2004), and models that deviate from it (Thompson, 2001; Akcigit and Kerr, 2018) or can deviate from it under specific parametrizations (Laincz, 2009; Acemoglu et al., 2018; Massari, 2023). They all have reasons to exist, as the former group presents cleaner and less mathematically challenging models. In contrast, the latter ones introduce models that are potentially more in line with the empirical evidence just discussed but are harder to handle. Therefore, a theorist is left wondering whether and for what purposes introducing this complication is worth it. Additionately, in the case of adopting models that deviate from Gibrat Law, knowing the correlation between firm growth rates and levels would provide a useful data moment for calibration. As economy-wide data to perform these decompositions are not publicly available, economists implementing them would offer a valuable service to applied theorists if they

[3] I focus exclusively on the dynamic Olley-Pakes decomposition because the result is more readily visible. The procedure to modify the Foster et al. (2001) decomposition is the same, but that decomposition features a more complex interaction between firms' productivities and their labor or market shares. The implication is that all its components are sensitive to the productivity definition, although the sign of the discrepancy cannot be determined a priori.

adopted the modification proposed in this paper.

The paper proceeds as follows: in section 2, I introduce a standard theoretical framework to derive the appropriate definitions of aggregate and average productivity; in section 3, I show that average productivity growth is sensitive to the definition of average; in section 4, I introduce the modification to the Dynamic Olley-Pakes decomposition; and in section 5, I conclude.

2 Defining Aggregate and Average Productivity

Productivity growth accounting relies on methodologies like Foster et al. (2001) or Melitz and Polanec (2015) that start from an arbitrary definition of aggregate productivity. The most widely used definition, as it yields the cleanest decompositions, is the weighted sum of firms' log productivity levels, with employment or market shares as exogenous weights. This definition implies a Cobb-Douglas aggregator where average output is the geometric mean of individual firms' output.

A related stream of literature (Osotimehin, 2019; Baqaee and Farhi, 2020) is concerned with the quantitative relevance for aggregate productivity of factors' misallocation due to various frictions. Those papers provide sufficient statistics to assess the contribution of factor reallocation in response to microeconomic shocks. Although these papers' purpose differs from productivity growth accounting, their proposed structure imposes more general and standard theoretical assumptions. Therefore, I rely on their simplest specification, a horizontal economy with a CES aggregator:

$$Y_t = \left[\int_0^{N_t} y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{1}$$

with N_t firms and firms' production function with only one input, labor:

$$y_{i,t} = A_{i,t} L_{i,t}^{\alpha}, \tag{2}$$

where $A_{i,t}$ is firm-level productivity.

Using equations 1 and 2 and multiplying and dividing by average productivity $A_t = S_{i,t} A_{i,t}$, average labor L_t/N_t, and the love of variety term, I obtain the following decomposition of aggregate labor productivity into average productivity and aggregation terms:

$$\frac{Y_t}{L_t} = \underbrace{A_t}_{\substack{\text{Average}\\\text{productivity}}} \underbrace{\underbrace{N_t^{\frac{1}{\epsilon-1}}}_{\substack{\text{Love of}\\\text{variety}}} \underbrace{\left\{\frac{1}{N_t}\int_0^{N_t} \left(\left(S_{i,t}N_t\right)^{\alpha} a_{i,t}\right)^{\frac{\epsilon-1}{\epsilon}} di\right\}^{\frac{\epsilon}{\epsilon-1}}}_{\text{Allocation}} \underbrace{\left(\frac{L_t}{N_t}\right)^{\alpha-1}}_{\substack{\text{Diminishing}\\\text{returns}}}}_{\text{Aggregation}}, \quad (3)$$

where $S_{i,t}$ is firm i's employment share, and $a_{i,t} = A_{i,t}/A_t$ is relative productivity.

The aggregation component collapses to 1 when the aggregator and production function are linear. Hence, these aggregation terms allow for the measurement of the effect on aggregate productivity that arises due to the presence of non-linearities. This pursuit is valuable as quantifying the magnitude of the aggregation component provides information regarding the benefit of introducing non-linearities in theoretical models.

Although commonly labeled aggregate productivity, the standard object of productivity growth accounting analysis is average productivity. The remainder of the paper focuses on this object. However, this definition is the average of productivity levels instead of the productivity log-levels (or, equivalently, geometric average of productivity levels) common in the productivity growth accounting literature. The next sections illustrate the implications of choosing one over the other for productivity growth measurements that start from firm-level data, and for productivity growth accounting decompositions.

3 Long-Run Growth under Different Definitions of Average Productivity

I compare the growth rate of average productivity based on two different definitions: the geometric average, more common in the literature, and the more theoretically appropriate arithmetic average. I prove that average productivity defined in these two different ways grows at different rates if firms' relative productivity levels are mean-reverting, as the empirical literature cited in the introduction suggests.

For the purpose of this section, I keep the proof simple and the argument more intuitive by assuming that the relative productivity distribution is time-invariant, which, under most theoretical frameworks, would imply a time-invariant firm-size distribution. In this way, I emphasize that this result does not depend on the weights attached to individual productivities. This assumption can be interpreted as a characterization of long-run equilibrium, as we do not observe nor expect a perpetual tendency towards monopoly or to perfect equality in firm sizes. Additionally, I disregard entry and exit (i.e., I assume a fixed number of firms), which would introduce a distraction that does not affect this result. The following proposition formalizes the result, and the rest of this section proves it.

Proposition 1 *If relative productivity across firms is mean-reverting, and, under a time-invariant distribution of relative productivities, the growth rate of the arithmetic average productivity of firms is lower than the growth rate of the geometric average by the degree of mean-reversion.*

Since the distribution is time-invariant, the proof relies on the following lemma.

Lemma 2 *Under a time-invariant distribution of relative productivity levels, the unweighted productivity average grows at the same rate as the weighted average.*

This lemma is proved by showing that weighted and unweighted average productivity have the same trend, making their ratio trendless. Dividing the unweighted average of productivities by the weighted average yields $\frac{\overline{A}_t}{A_t} = \frac{1}{N_t}\int_0^{N_t} a_{i,t} di$, which is trendless when the relative productivity distribution is time-invariant, as assumed.

At this point, the growth rate of unweighted average productivity is:

$$\frac{\overline{A}_t - \overline{A}_{t-1}}{\overline{A}_{t-1}} = \frac{1}{N_t}\int_0^{N_t} \frac{A_{i,t-1}}{\overline{A}_{t-1}} g_t^{A_i}. \tag{4}$$

Because $\int_0^{N_t} \frac{1}{N_t}\left(A_{i,t-1}/\overline{A}_{t-1} - 1\right) di = 0$, the equation can be re-expressed as:

$$g_t^{\overline{A}} = \underbrace{\overline{g}_t^{A_i}}_{\text{Average growth}} + \underbrace{cov\left(g_t^{A_i}, \overline{a}_{i,t-1}\right)}_{\text{Churning}}, \tag{5}$$

where the first term is the simple arithmetic average of firms' productivity growth rate, and $\overline{a}_{i,t} = \frac{A_{i,t}}{\overline{A}_t}$.

It is easy to verify that, under the approximation that time differences in natural logarithms equal growth rates, the growth rate of the geometric average of productivities equals the average of the growth rates. Instead, notice that equation (5) includes an extra term, labeled *churning*, which is a covariance of productivity growth rates and levels. By churning, I mean the movement of firms within the relative productivity distribution, which occurs even when this distribution is time-invariant, implying that this phenomenon is also present in a steady state. The importance of this component is exemplified by the existence and success of the literature on firm dynamics based primarily on Hopenhayn (1992), where any phenomenon analyzed derives from exogenous idiosyncratic shocks to firms' productivity levels. Within that framework, the churning component determines how persistent these shocks are. Equation (5) shows that this persistence directly affects the steady-state growth rate of average productivity, as both churning and average productivity growth are exogenous in firm dynamics models.

4 Revisiting the Dynamic Olley-Pakes Decomposition

In this section, I revisit the dynamic Olley-Pakes decomposition proposed in Melitz and Polanec (2015), motivated by the results obtained in the previous two sections. First, I have established that the weighted arithmetic average is a more appropriate definition of average productivity. Then, I have shown that, under the arithmetic average definition, a covariance between productivity growth and levels accounts for the difference between the growth rate of average productivity and the average of the growth rates. This covariance must, therefore, show up somewhere in any decomposition. In what follows, I show where.

The dynamic Olley-Pakes decomposition decomposes average productivity change into an unweighted average change (within component), a change in covariance between employment shares and firm-level productivity (between component), and entry/exit:

$$A_t - A_{t-1} = \underbrace{\int_0^{N_{c_{t-1}}} \frac{A_{i,t} - A_{i,t-1}}{N_{c_{t-1}}} di}_{\text{Within}} + \underbrace{\Delta cov_{c_t}}_{\text{Between}} + \underbrace{S_{E_t}\left(A_{E_t} - A_{c_t}\right)}_{\text{Entry}} + \underbrace{S_{X_t}\left(A_{c_{t-1}} - A_{X_{t-1}}\right)}_{\text{Exit}}, \quad (6)$$

where

$$\Delta cov_{c_t} = \int_0^{N_{c_{t-1}}} \left(\frac{S_{i,t}}{S_{c_t}} - \frac{1}{N_{c_{t-1}}} \right) \left(A_{i,t} - A_{c_t} \right) di - \int_0^{N_{c_{t-1}}} \left(\frac{S_{i,t-1}}{S_{c_{t-1}}} - \frac{1}{N_{c_{t-1}}} \right) \left(A_{i,t-1} - A_{c_{t-1}} \right) di. \quad (7)$$

Dividing both sides by A_{t-1} yields:

$$g_t^A = \underbrace{\int_0^{N_{c_{t-1}}} \frac{1}{N_{c_{t-1}}} a_{i,t-1}\, g_t^{A_i}}_{\text{Within}} + \underbrace{\frac{\Delta cov_{c_t}}{A_t}}_{\text{Between}} + \underbrace{S_{E_t} \frac{A_t}{A_{t-1}} \left(a_{E_t} - a_{c_t} \right)}_{\text{Entry}} + \underbrace{S_{X_{t-1}} \left(a_{c_{t-1}} - a_{X_{t-1}} \right)}_{\text{Exit}}, \quad (8)$$

where $a_{c_t} = \frac{A_{c_t}}{A_t}$, $a_{E_t} = \frac{A_{E_t}}{A_t}$, $a_{X_t} = \frac{A_{X_t}}{A_t}$.

Notice the difference in the within component of equations 6 and 8. When average productivity is a geometric mean, productivity in equation 6 is in log levels. Thus the equation decomposes the growth rate of average productivity. In that case, the within component is an unweighted arithmetic average of firms' productivity growth rates. When average productivity is an arithmetic mean, the relevant decomposition is equation 8, where the within component is a weighted average of firms' growth rates, with the relative productivity level as the weight.

To see the implications of this result, I decompose the within component further by dividing it in two parts:

$$g_t^A = \underbrace{\underbrace{\bar{g}_t^{A_i}}_{\text{Average growth}} + \underbrace{\int_0^{N_{c_{t-1}}} \frac{a_{i,t-1} - 1}{N_{c_{t-1}}} g_t^{A_i} di}_{\text{Heterogeneity}}}_{\text{Within}} \quad (9)$$

where $\bar{g}_t^{A_i} = \int_0^{N_{c_{t-1}}} \frac{1}{N_{c_{t-1}}} g_t^{A_i} di$ represents the unweighted average of the productivity growth rates of individual continuing firms.

Using the fact that $\int_0^{N_{c_{t-1}}} \frac{1}{N_{c_{t-1}}} \left(\bar{a}_{i,t-1} - \bar{a}_{c_{t-1}} \right) di = 0$, where $\bar{a}_{c_t} = \frac{\bar{A}_{c_t}}{\bar{A}_t}$, and $\overline{A}_{i,t} = \int_0^{N_t} \frac{A_{i,t}}{N_t} di$, the heterogeneity component is:

$$g_t^{HETEROGENEITY} = \underbrace{\left(\overline{a}_{c_{t-1}} - 1\right) \overline{g}_t^{A_i}}_{\text{Survival selection}} + \underbrace{cov\left(g_t^{A_i}, \overline{a}_{i,t-1}\right)}_{\text{Churning}}. \tag{10}$$

Survival selection is a correction term. It depends on the relative productivity level of surviving firms and whether surviving firms are, on average, more or less productive than exiting firms. The higher the relative productivity of survivors, the higher the contribution to aggregate growth from this channel. Models that assume an exogenous exit shock that hits all firms symmetrically would miss this. Hence, the size of this sub-component can inform theorists about the benefit of modeling exit endogenously.

Churning is the same covariance between relative productivity levels and absolute productivity growth rates across firms identified in the previous section. It describes the nature of firm mobility within the relative productivity distribution. This sub-component is interesting for theoretical purposes. In addition to the firm dynamics literature discussed in the previous section, this component also offers valuable information for growth theorists. Growth models that deliver Gibrat Law, like Klette and Kortum (2004) and its derivatives, predict that this sub-component is 0. Theories that deviate from Gibrat Law (Thompson, 2001; Akcigit and Kerr, 2018), or can deviate from it under some parametrizations (Acemoglu et al., 2018; Massari, 2023), predict a negative sign. Theories like Laincz (2009) predict a positive sign. Importantly, this component would be absent if average productivity was defined as a geometric average, leading to an overestimation of the within component in case of a negative covariance.

5 Conclusion

This paper analyzes the implications of different average productivity definitions in the context of productivity growth accounting. It proves that, fixing firms' growth rates, the geometric average grows faster than the arithmetic average when productivity growth and levels are negatively correlated. Then, it modifies the popular dynamic Olley-Pakes decomposition to emphasize the component responsible for this discrepancy. The modification is justified on the grounds that this component offers valuable insights to applied theorists, as it can guide their model choice and calibration.

Therefore, I encourage analysts with access to private, economy-wide data on firms' productivities and factor shares to adopt a definition of average productivity as the weighted arithmetic average of individual productivities. Moreover, I encourage them to decompose the within component further and report the covariance between productivity growth rates and levels.

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