Revisiting Productivity Growth Accounting Decompositions

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Abstract

This paper shows the conditions under which productivity growth accounting decompositions are sensitive to the definition of aggregate productivity and why they matter. Following standard theoretical assumptions, it proposes (i) a distinction between aggregate and average productivity and (ii) using the arithmetic mean instead of the more common geometric mean when defining average productivity.

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1 Introduction

What explains changes in aggregate productivity? Economists, practitioners, and policy-makers employ productivity growth accounting techniques to decompose aggregate productivity as a first diagnostic tool to guide further analysis. These decompositions illustrate how much of aggregate productivity growth is associated with improved firm-level productivity or reallocation of resources across incumbents, entering, and exiting firms.[1] Furthermore, these decompositions allow determining moments from the data helpful in calibrating or selecting the most appropriate models (Bartelsman et al., 2013).

This paper points out that, whenever firms' productivity levels correlate with their growth rates, the aggregate productivity growth rate and its decomposition are sensitive to the aggregate productivity definition. Athough Melitz and Polanec (2015) note that results change based on the definition, I go a step further by showing that (i) results differ by a data moment that is of importance for theoretical purposes, and (ii) one definition ought to be preferred on theoretical grounds over the most widely used. In particular, if productivity levels and growth rates are negatively correlated, the commonly used definition overstates productivity growth and misattributes part of it to the within-firm component. This result is relevant because the literature documents both a positive association between firm or

[1] Some recent and valuable examples of how these decompositions are employed include an assessment of productivity dynamics during the Covid pandemic (Bloom et al., 2023), making sense of sluggish Italian productivity growth (Bugamelli et al., 2018), understanding the nature of Chinese productivity growth Gao et al. (2023).

establishment employment and productivity (Bartelsman et al., 2013), and a negative correlation between employment growth rates and levels — see, for example, Rossi-Hansberg and Wright (2007). Therefore, adopting the more appropriate definition would inform theorists on the sign and magnitude of data moments that could be useful for choosing and calibrating models.

2 Defining Aggregate and Average Productivity

Productivity growth accounting relies on methodologies like Foster et al. (2001) or Melitz and Polanec (2015) that start from an arbitrary definition of aggregate productivity. The most widely used definition, as it yields the cleanest decomposition, is the weighted sum of firms' log productivity levels, with employment or market shares as exogenous weights. This definition implies a Cobb-Douglas aggregator where average output is the geometric mean of individual firms' output.

A related stream of literature (Osotimehin, 2019; Baqaee and Farhi, 2020) is concerned with the quantitative relevance for aggregate productivity of factors' misallocation due to various frictions. Those papers provide sufficient statistics to assess the contribution of factor reallocation in response to microeconomic shocks. Although these papers' purpose differs from productivity growth accounting, the structure that they propose imposes more general and standard theoretical assumptions. I, therefore, rely on their horizontal economy framework with a CES aggregator:

$$Y_t = \left[\int_0^{N_t} y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{1}$$

with N_t firms and firms' production function with only one input, labor:

$$y_{i,t} = A_{i,t} L_{i,t}^{\alpha}, \tag{2}$$

where $A_{i,t}$ is firm-level productivity.

Using equations 1 and 2 and multiplying and dividing by average productivity $A_t = S_{i,t} A_{i,t}$, average labor L_t / N_t, and the love of variety term, I obtain the following decomposition of aggregate labor productivity into average productivity and aggregation terms:

$$\frac{Y_t}{L_t} = \underbrace{A_t}_{\substack{\text{Average}\\ \text{productivity}}} \underbrace{\underbrace{N_t^{\frac{1}{\epsilon-1}}}_{\substack{\text{Love of}\\ \text{variety}}} \underbrace{\left\{ \frac{1}{N_t} \int_0^{N_t} \left((S_{i,t} N_t)^{\alpha} a_{i,t} \right)^{\frac{\epsilon-1}{\epsilon}} di \right\}^{\frac{\epsilon}{\epsilon-1}}}_{\text{Allocation}} \underbrace{\left(\frac{L_t}{N_t} \right)^{\alpha-1}}_{\substack{\text{Diminishing}\\ \text{returns}}}}_{\text{Aggregation}}, \tag{3}$$

where $S_{i,t}$ is firm i's employment share, and $a_{i,t} = A_{i,t} / A_t$ is relative productivity.

The aggregation component collapses to 1 when the aggregator and production function are linear. Hence, these aggregation terms allow for the measurement of the effect on aggregate productivity that arises due to the presence of non-linearities. This pursuit is valuable as quantifying the magnitude of the aggregation component provides information regarding the benefit of introducing non-linearities in theoretical models.

Although commonly called aggregate productivity, the standard object of productivity growth accounting analysis is average productivity. However, this definition is the average of productivity levels instead of the productivity log levels common in the productivity growth accounting literature. The following section illustrates the implications of choosing one over the other for productivity growth accounting decompositions.

3 Long-Run Growth under Different Definitions of Average Productivity

I compare the growth rate of average productivity based on two different definitions: the geometric average, more common in the literature, and the more theoretically appropriate arithmetic average. I prove that average productivity defined in these two different ways grows at different rates if firms' relative productivity levels are mean-reverting. To keep the proof simple and the argument more intuitive, I will assume that the relative productivity distribution is time-invariant, which, under most theoretical frameworks, would imply a time-invariant firm-size distribution. In this way, I emphasize that this result does not depend on the weights attached to individual productivities. The following proposition formalizes the result, and the rest of this subsection proves it.

Proposition 1 *If relative productivity across firms is mean-reverting, and, under a time-invariant distribution of relative productivities, the growth rate of the arith-*

metic average productivity of continuing firms is lower than the growth rate of the geometric average by the degree of mean-reversion (after correcting for entry and exit effects).

Since the distribution is time-invariant, I rely on the following lemma.

Lemma 2 *Under a time-invariant distribution of relative productivity levels, the unweighted productivity average grows at the same rate as the weighted average.*

This lemma is proved by showing that weighted and unweighted average productivity have the same trend, making their ratio trendless. Dividing the unweighted average of productivities by the weighted average yields $\frac{\overline{A}_t}{A_t} = \frac{1}{N_t}\int_0^{N_t} a_{i,t} di$, which is trendless when the relative productivity distribution is time-invariant.

At this point, the dynamic Olley-Pakes decomposition proposed by Melitz and Polanec (2015) is handy.[2] It decomposes average productivity change into an unweighted average change (within component), a change in covariance between employment shares and firm-level productivity (between component), and entry/exit:

$$A_t - A_{t-1} = \underbrace{\int_0^{N_{c_{t-1}}} \frac{A_{i,t} - A_{i,t-1}}{N_{c_{t-1}}} di}_{\text{Within}} + \underbrace{\Delta cov_{c_t}}_{\text{Between}} + \underbrace{S_{E_t}\left(A_{E_t} - A_{c_t}\right)}_{\text{Entry}} + \underbrace{S_{X_t}\left(A_{c_{t-1}} - A_{X_{t-1}}\right)}_{\text{Exit}}, \quad (4)$$

[2]Relying on the Foster et al. (2001) decomposition would lead to similar insights, although the derivation is less straightforward.

where

$$\Delta cov_{c_t} = \int_0^{N_{c_{t-1}}} \left(\frac{S_{i,t}}{S_{c_t}} - \frac{1}{N_{c_{t-1}}} \right) \left(A_{i,t} - A_{c_t} \right) di - \int_0^{N_{c_{t-1}}} \left(\frac{S_{i,t-1}}{S_{c_{t-1}}} - \frac{1}{N_{c_{t-1}}} \right) \left(A_{i,t-1} - A_{c_{t-1}} \right) di. \quad (5)$$

Dividing both sides by A_{t-1} yields:

$$g_t^A = \underbrace{\int_0^{N_{c_{t-1}}} \frac{1}{N_{c_{t-1}}} a_{i,t-1}\, g_t^{A_i}}_{\text{Within}} + \underbrace{\frac{\Delta cov_{c_t}}{A_t}}_{\text{Between}} + \underbrace{S_{E_t} \frac{A_t}{A_{t-1}} \left(a_{E_t} - a_{c_t} \right)}_{\text{Entry}} + \underbrace{S_{X_{t-1}} \left(a_{c_{t-1}} - a_{X_{t-1}} \right)}_{\text{Exit}}, \quad (6)$$

where $a_{c_t} = \frac{A_{c_t}}{A_t}$, $a_{E_t} = \frac{A_{E_t}}{A_t}$, $a_{X_t} = \frac{A_{X_t}}{A_t}$.

Notice the difference in the within component of equations 4 and 6. When average productivity is a geometric mean, productivity in equation 4 is in log levels. Thus the equation decomposes the growth rate of average productivity. The within component is an average of firms' growth rates. When average productivity is an arithmetic mean, the relevant decomposition is equation 6, where the within component is a weighted average of firms' growth rates, with the relative productivity level as the weight.

To see the implications of this result, I decompose the within component further by dividing it in two parts:

$$g_t^A = \underbrace{\underbrace{\overline{g}_t^{A_i}}_{\text{Average growth}} + \underbrace{\int_0^{N_{c_{t-1}}} \frac{a_{i,t-1} - 1}{N_{c_{t-1}}} g_t^{A_i} di}_{\text{Heterogeneity}}}_{\text{Within}} \tag{7}$$

where $\overline{g}_t^{A_i} = \int_0^{N_{c_{t-1}}} \frac{1}{N_{c_{t-1}}} g_t^{A_i} di$ represents the unweighted average of the productivity growth rates of individual continuing firms.

Using the fact that $\int_0^{N_{c_{t-1}}} \frac{1}{N_{c_{t-1}}} \left(\overline{a}_{i,t-1} - \overline{a}_{c_{t-1}} \right) di = 0$, where $\overline{a}_{i,t} = \frac{A_{i,t}}{\overline{A}_t}$ and $\overline{a}_{c_t} = \frac{\overline{A}_{c_t}}{\overline{A}_t}$, and $\overline{A}_{i,t} = \int_0^{N_t} \frac{A_{i,t}}{N_t} di$, the heterogeneity component is:

$$g_t^{HETEROGENEITY} = \underbrace{\left(\overline{a}_{c_{t-1}} - 1 \right) \overline{g}_t^{A_i}}_{\text{Survival selection}} + \underbrace{cov \left(g_t^{A_i}, \overline{a}_{i,t-1} \right)}_{\text{Systematic churning}}. \tag{8}$$

Survival selection is a correction term. It depends on the relative productivity level of surviving firms and on whether surviving firms are, on average, more or less productive than exiting firms. The higher the relative productivity of survivors, the higher the contribution to aggregate growth from this channel. Models that assume an exogenous exit shock that hits all firms symmetrically would miss this. Hence, the size of this sub-component can inform theorists about the benefit of modeling exit endogenously.

Systematic churning is a covariance between relative productivity level and absolute productivity growth rates across firms. It describes the nature of firm mobility within the relative productivity distribution. This sub-component is interesting for theoretical purposes. Models that deliver

Gibrat Law, like Klette and Kortum (2004) and its derivatives, predict that this sub-component is 0.[3] Theories that deviate from Gibrat Law, like Thompson (2001) or (Massari, 2023) under some parametrizations, predict a negative sign. Furthermore, this component would be absent if average productivity was defined as a geometric average, leading to an overestimation of the within component.

4 Conclusion

This paper started defining aggregate labor productivity based on standard theoretical assumptions. It then decomposed aggregate productivity into an average productivity level and an aggregation component. This distinction is helpful because it allows assessing the quantitative relevance of non-linearities. Furthermore, it proved that average productivity growth and its decomposition are sensitive to its definition only when productivity growth rates correlate with their relative levels. Although the geometric average tends to be preferred as it leads to a mathematically cleaner decomposition, the arithmetic average has a stronger theoretical justification and includes additional moments that are useful to discriminate between different theoretical models and calibrate them.

[3] Gibrat Law is the statement that firm sizes and their proportional growth rates are uncorrelated. See Sutton (1997) for a discussion of the empirical evidence rejecting it.

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