Online Appendix to "Current Account and Trade Balance Dynamics in a Schumpeterian Small Open Economy"

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This Online Appendix accompanies the submission version of the paper. It collects the household and firm microfoundations, the detailed characterization of the Schumpeterian block, and the derivations that are summarized in the main text.

1 Household Problem

We embed the Peretto–Smulders Schumpeterian framework of innovation and firm dynamics, as articulated in Peretto (2026) and Chu and Peretto (2026), in a small open economy populated by individual households, each with a finite expected lifetime, in the tradition of Blanchard (1985) and Yaari (1965).

An individual born at time i has utility

$$U_i = \int_0^\infty e^{-(\rho+\delta)t} \left[\log\left(\frac{E_i}{p_Y}\right) + \mu \log\left(\frac{M_i}{p_m}\right) \right] dt, \tag{A1}$$

where E_i is expenditure on the domestic good, p_Y is the price of the domestic good, $\mu > 0$ is a preference parameter for the imported good, M_i is expenditure on imports, and p_m is the import price. The individual's dynamic budget constraint is

$$\dot{A}_i = rA_i + w - E_i - (1+\tau)M_i + T_i, \tag{A2}$$

where A_i is individual wealth, w is the wage, τ is an ad valorem tariff on imports, and T_i is a lump-sum transfer.

Dynamic optimization yields expenditure on the imported good,

$$M_i = \frac{\mu E_i}{1+\tau}, \tag{A3}$$

as well as the individual Euler equation $r = \rho + \delta + \dot{E}_i/E_i$. Aggregation across cohorts gives the aggregate Euler equation

$$\frac{\dot{E}}{E} = r - \rho + \lambda - (\lambda+\delta)(\rho+\delta)\frac{A}{E}, \tag{A4}$$

where ρ is the time discount rate, δ is the probability of death, and λ is population growth. The wealth term reflects the distribution of financial wealth across cohorts under finite lives, as in Obstfeld and Rogoff (1996). This is the key demand-side equation used in the main text.

Aggregate imports satisfy

$$M = \int_{-\infty}^{t} \chi_i M_i \, di = \frac{\mu}{1+\tau} E, \tag{A5}$$

so the ratio of aggregate imports to domestic expenditure is constant and decreasing in the tariff. The government's tariff revenue is τM, rebated to households in lump-sum form. The country's aggregate budget constraint becomes

$$\dot{A} = rA + (1-\theta)p_Y Y - \frac{1+\tau+\mu}{1+\tau}E. \tag{A6}$$

2 Production, Innovation, and Entry

The final-good sector is perfectly competitive and generates zero profit. Following Peretto (2015), the production technology is

$$Y = \int_0^N X_i^\theta \left(Z_i^\alpha Z^{1-\alpha} \frac{L}{N^{1-\sigma}} \right)^{1-\theta} di, \qquad \theta, \alpha, \sigma \in (0,1), \tag{A7}$$

where N is the mass of intermediate varieties, X_i is the quantity of intermediate good i, and Z is average quality. Profit maximization yields the labor demand condition

$$L = (1-\theta)\frac{p_Y Y}{w}, \tag{A8}$$

and the demand system for intermediate goods,

$$X_i = \left(\frac{p_Y \theta}{p_{X_i}} \right)^{\frac{1}{1-\theta}} \frac{Z_i^\alpha Z^{1-\alpha} L}{N^{1-\sigma}}. \tag{A9}$$

Each monopolistic intermediate firm produces the intermediate good one-for-one with the final good, pays fixed operating cost $\phi Z_i^\alpha Z^{1-\alpha}$, and invests I_i units of the final good in in-house R&D:

$$\dot{Z}_i = I_i. \tag{A10}$$

The firm's profit before R&D is

$$\Pi_i = (p_{X_i} - p_Y)X_i - p_Y \phi Z_i^\alpha Z^{1-\alpha}, \tag{A11}$$

and firm value is

$$V_i = \int_t^\infty \exp\left(-\int_t^s r_u \, du \right) [\Pi_i(s) - p_Y I_i(s)] \, ds. \tag{A12}$$

Markup pricing implies $p_{X_i} = \eta p_Y$, and the internal rate of return to in-house R&D is

$$r = \alpha \left[(\eta - 1) \left(\frac{p_Y \theta}{p_{X_i}} \right)^{\frac{1}{1-\theta}} \frac{L}{N^{1-\sigma}} - \phi \right] \left(\frac{Z}{Z_i} \right)^{1-\alpha} + \frac{\dot{p}_Y}{p_Y}. \tag{A13}$$

A new firm pays an entry cost of $\beta\theta Y/N$ to create a new variety. Free entry requires

$$V = \beta\theta \frac{p_Y Y}{N}. \tag{A14}$$

Rewriting the asset-pricing equation in return form yields

$$r = \frac{\Pi_i}{V_i} + \frac{\dot{V}_i}{V_i}. \tag{A15}$$

The rate of return to entry is therefore

$$r = \frac{1}{\beta\theta \frac{p_Y Y}{N}} \left[(\eta - 1) \left(\frac{p_Y \theta}{p_{X_i}} \right)^{\frac{1}{1-\theta}} \frac{L}{N^{1-\sigma}} - \phi \right] Z_i^{\alpha} Z^{1-\alpha} + \frac{\dot{p}_Y}{p_Y} + \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N}. \tag{A16}$$

3 Symmetric Equilibrium and the Schumpeterian Block

As in Peretto (1998, 1999, 2015, 2026), when firms start from the same stock of knowledge the intermediate-sector equilibrium is symmetric: $Z_i = Z$, $X_i = X$, $\Pi_i = \Pi$, and $V_i = V$. In symmetric equilibrium, the reduced-form production function is

$$Y = \left(\frac{\theta p_Y}{p_X} \right)^{\frac{\theta}{1-\theta}} N^{\sigma} Z L, \tag{A17}$$

and quality-adjusted firm size is

$$x \equiv \frac{X}{Z} = \left(\frac{p_Y \theta}{p_X} \right)^{\frac{1}{1-\theta}} \frac{L}{N^{1-\sigma}}. \tag{A18}$$

Define the innovation and entry rates as $z \equiv \dot{Z}/Z$ and $n \equiv \dot{N}/N$. The two return equations become

$$r = \alpha \left[(\eta - 1)x - \phi \right] + \frac{\dot{p}_Y}{p_Y}, \tag{A19}$$

$$r = \frac{1}{\beta\theta} \left(\eta - 1 - \frac{\phi + z}{x} \right) + \frac{\dot{p}_Y}{p_Y} + \frac{\dot{Y}}{Y} - n. \tag{A20}$$

The return to entry implies the threshold

$$x_N = \frac{\phi}{\eta - 1}, \tag{A21}$$

and there exists a second threshold $x_Z > x_N$ above which in-house innovation is active. The entry rate is piecewise:

$$n(x) = \begin{cases} 0, & 0 < x \le x_N, \\ \dfrac{\eta - 1}{\beta\theta} - r - \dfrac{\phi}{\beta\theta x}, & x_N < x \le x_Z, \\ \dfrac{\eta - 1}{\beta\theta} - r + \dfrac{(1-\alpha)\phi - (1-\sigma)r + \alpha(\eta-1)x - \dfrac{\sigma(\eta-1)}{\beta\theta}}{\sigma - \beta\theta x}, & x_Z < x < \infty. \end{cases} \tag{A22}$$

Hence market-structure dynamics are summarized by

$$\frac{\dot{x}}{x} = \lambda - (1-\sigma)n(x). \tag{A23}$$

Global stability of the scale-invariant endogenous-growth steady state follows under the endogenous growth condition

$$\frac{(1-\alpha)\phi - \left(r + \dfrac{\lambda}{1-\sigma}\right)}{(1-\alpha)(\eta-1) - \beta\theta\left(r + \dfrac{\lambda}{1-\sigma}\right)} > \frac{\phi}{\eta - 1 - \beta\theta r}. \tag{A24}$$

The growth rate is

$$g = \sigma n + z, \tag{A25}$$

and in the active-innovation region it reduces to

$$g = \alpha\big[(\eta-1)x - \phi\big] - r. \tag{A26}$$

The steady-state firm size is

$$x^* = \frac{(1-\alpha)\phi - \left(r + \dfrac{\lambda}{1-\sigma}\right)}{(1-\alpha)(\eta-1) - \beta\theta\left(r + \dfrac{\lambda}{1-\sigma}\right)}, \tag{A27}$$

and the associated growth rate is

$$g^* = \alpha\big[(\eta-1)x^* - \phi\big] - r. \tag{A28}$$

Simple algebra gives

$$\frac{dx^*}{dr} \lesseqgtr 0 \qquad \text{for} \qquad \beta\theta\phi \gtreqless \eta - 1. \tag{A29}$$

4 International Accounting Derivation

A cornerstone of international macroeconomics is the identity

$$CA = TB + rB. \tag{A30}$$

Starting from the aggregate household budget constraint,

$$\dot{A} = rA + wL - E - M, \tag{A31}$$

and decomposing wealth as $A = NV + B$, we obtain

$$\dot{D} + \dot{B} = rD + rB + wL - E - M, \tag{A32}$$

with $D \equiv NV$. Since domestic assets are equity created by home agents when they create new intermediate firms, we can write

$$\dot{N}V + N\dot{V} + \dot{B} = \left(\frac{\Pi}{V} + \frac{\dot{V}}{V}\right) NV + rB + wL - E - M. \tag{A33}$$

This reduces to

$$\dot{N}V + \dot{B} = N\Pi + rB + wL - E - M. \tag{A34}$$

Expanding intermediate-goods profits yields

$$\dot{N}V + \dot{B} = N(p_X - p_Y)X - Np_Y\phi Z - Np_Y I + rB + wL - E - M. \tag{A35}$$

Using the intermediate-goods profit definition together with the final-good producer's factor payments gives

$$\dot{B} = rB + TB. \tag{A36}$$

When the interest rate is constant, integrating forward under the transversality condition delivers the present-value relationship between the initial foreign asset position and the discounted stream of trade balances.

5 Canonical Intertemporal Approach: Additional Derivations

In the canonical approach, wealth is $A = \beta\theta p_Y Y + B$. Setting $p_Y Y = 1$ gives the dynamical system

$$\dot{E} = (r - \rho + \lambda)E - (\lambda + \delta)(\rho + \delta)(\beta\theta + B), \tag{A37}$$

$$\dot{B} = rB + r\beta\theta + 1 - \theta - \frac{1 + \tau + \mu}{1 + \tau}E. \tag{A38}$$

Under saddle-path stability, the foreign asset position has the closed-form solution

$$B(t) = B^* + \left(B_0 - B^*\right)e^{k_1 t}, \qquad k_1 < 0. \tag{A39}$$

This expression underlies the tariff results discussed in the main text. In particular, a higher tariff shifts down the entire path of the foreign asset position. Consequently, independently of whether the economy converges from above or below, a tariff increase moves the economy onto a lower path for external assets.

6 Valuation Approach: Additional Derivations

In the valuation approach, exports are explicit. Starting from

$$TB = Q - M = p_Y Y - \left[p_Y N(X + \phi Z + I) + \dot{N}V + E\right] - M, \tag{A40}$$

and using the normalization $p_Y Y = 1$, the export identity becomes

$$Q \equiv p_Y Y \left[1 - \frac{\theta}{\eta} - F(x)\right] - E, \tag{A41}$$

with

$$F(x) \equiv \frac{\phi + z(x)}{x} + n(x). \tag{A42}$$

Equivalently,

$$\frac{Q}{p_Y} + \frac{E}{p_Y} + F(x)Y = Y\left(1 - \frac{\theta}{\eta}\right). \tag{A43}$$

The constrained expenditure locus is therefore

$$E = 1 - \frac{\theta}{\eta} - F(x) - Q. \tag{A44}$$

Substituting into the accounting block yields

$$\dot{B} = rB + Q - \frac{\mu}{1+\tau}\left[1 - \frac{\theta}{\eta} - F(x)\right], \tag{A45}$$

$$\dot{x} = \lambda - (1-\sigma)n(x). \tag{A46}$$

The associated $\dot{B} = 0$ locus is

$$B \geq \frac{1}{r}\left[\frac{\mu}{1+\tau}\left(1 - \frac{\theta}{\eta} - F(x)\right) - Q\right]. \tag{A47}$$

Since the eigenvalue associated with B is positive, the foreign asset position must jump to the saddle path. The steady-state foreign asset position is

$$B^* = \frac{1}{r}\left[\frac{\mu}{1+\tau}\left(1 - \frac{\theta}{\eta} - F(x^*)\right) - Q\right] = -\frac{TB^*}{r}. \tag{A48}$$

An increase in export demand lowers the $\dot{B} = 0$ locus and produces an immediate downward jump in the foreign asset position. Interest-rate shocks affect both B and x through the sign of dx^*/dr. When $\beta\theta\phi < \eta - 1$, a higher interest rate lowers steady-state firm size; when $\beta\theta\phi > \eta - 1$, it raises steady-state firm size.

7 Figures

For convenience, this appendix reproduces the original figures individually.

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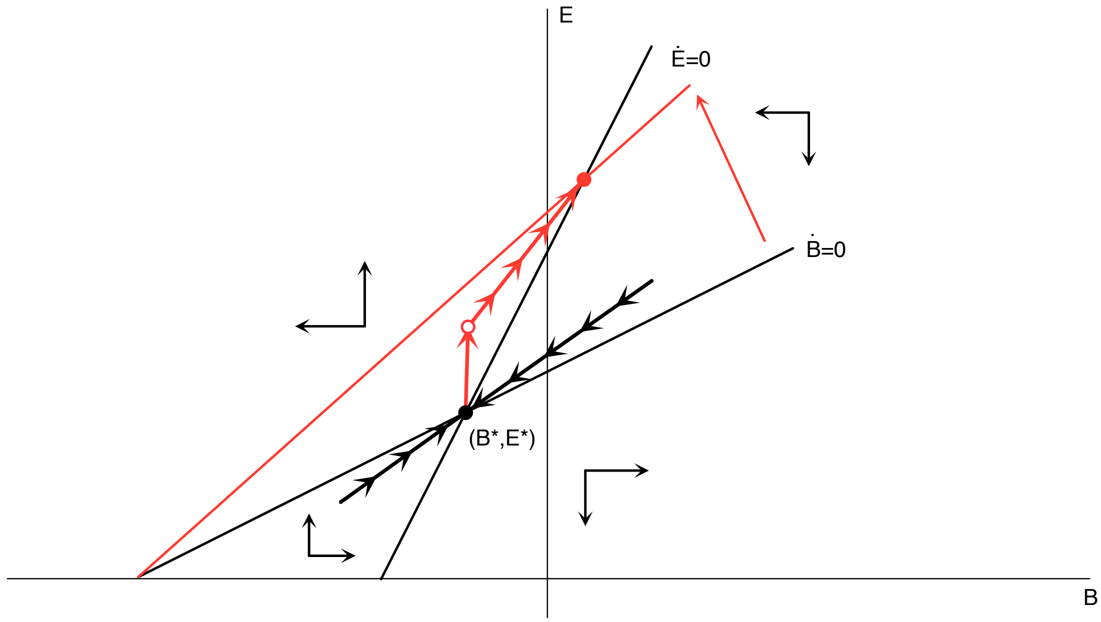

Figure A1: SOE OLG model: dynamics in (B, E) space, with the impulse response to the introduction of the tariff shown in red.

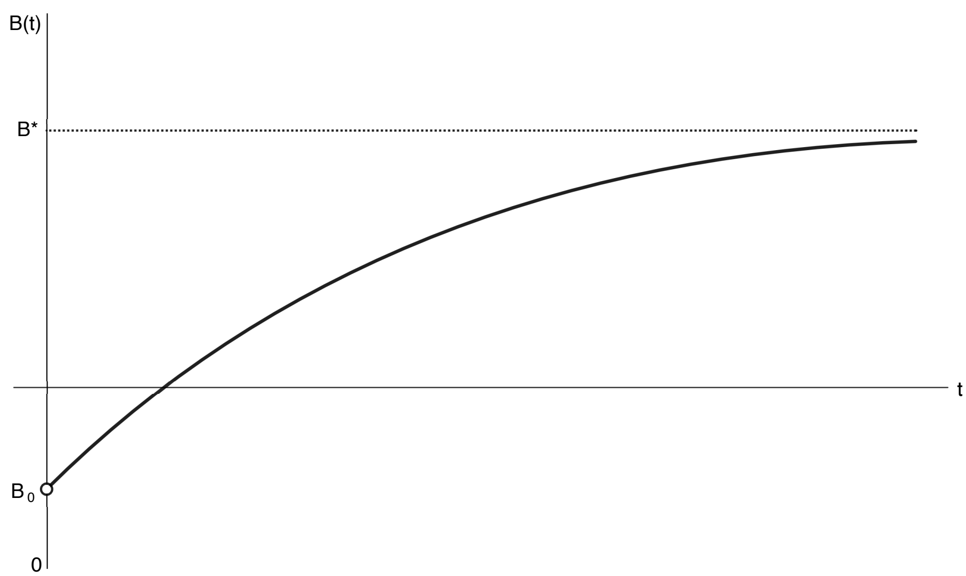

Figure A2: Transition of the SOE to the new steady state after the introduction of the tariff.

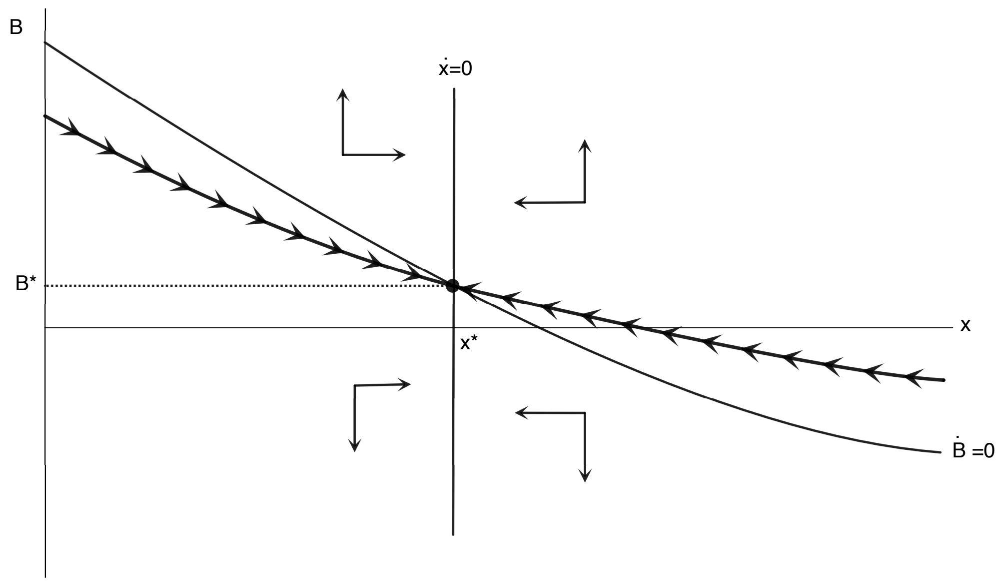

Figure A3: Valuation approach: equilibrium dynamics.

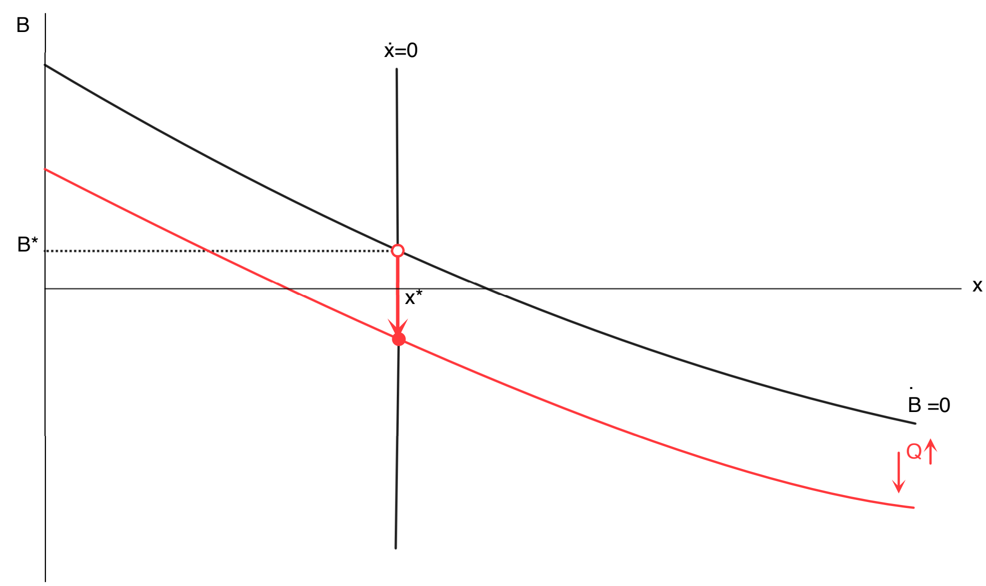

Figure A4: Valuation approach: effects of an increase in exports.

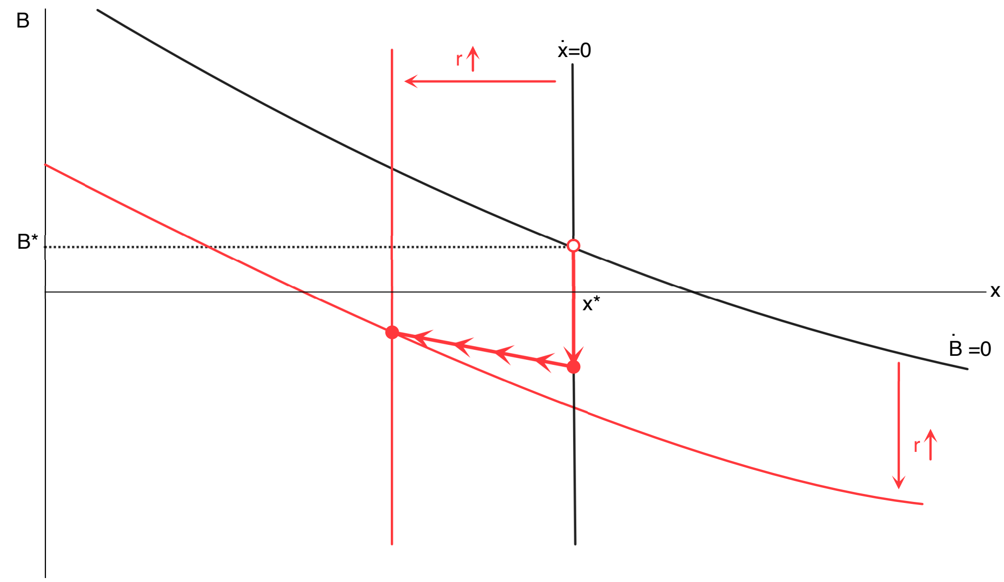

Figure A5: Valuation approach: effects of an increase in the world interest rate when higher r lowers steady-state firm size.

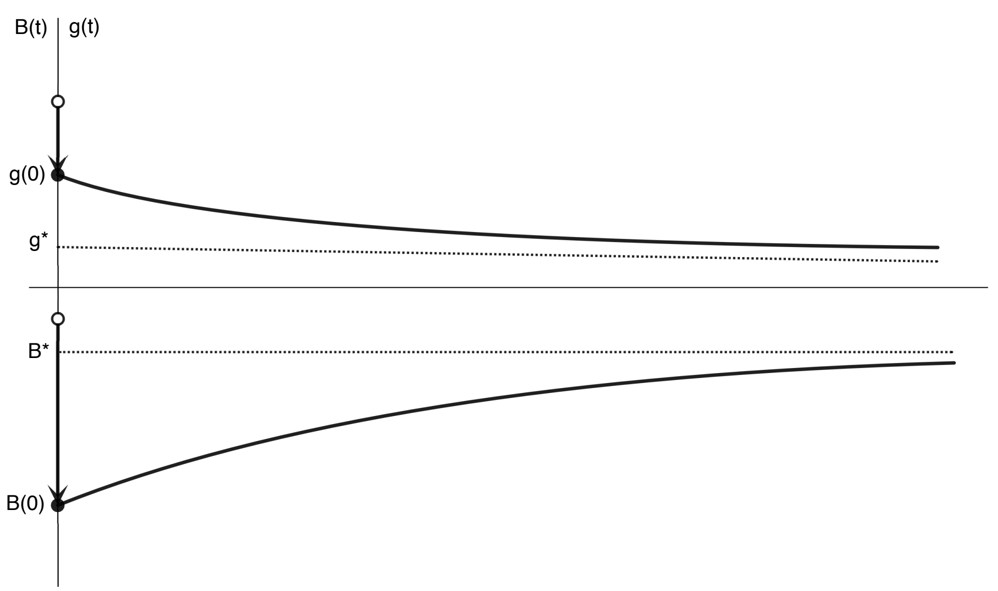

Figure A6: Valuation approach: impulse responses of growth and the foreign asset position when higher r lowers steady-state firm size.

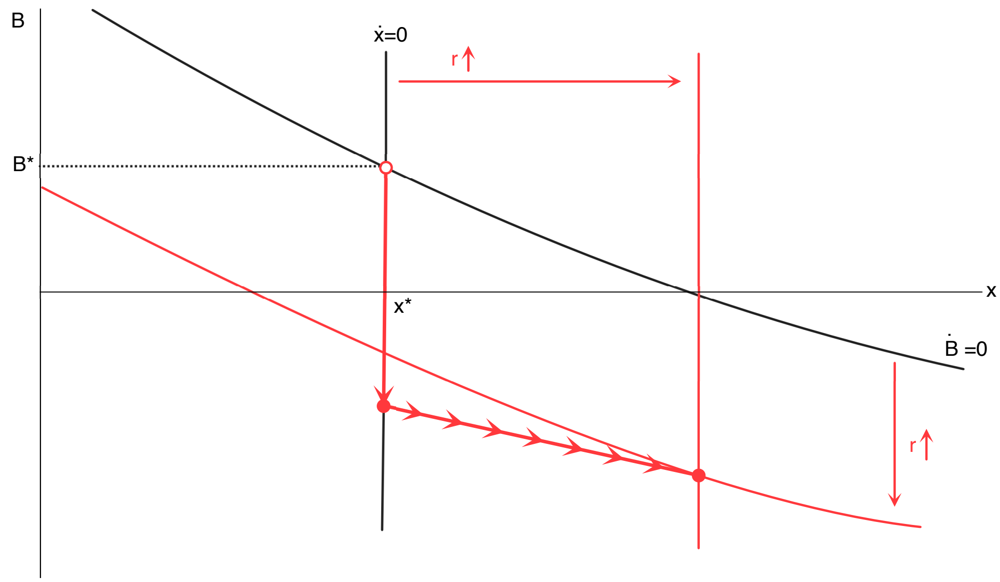

Figure A7: Valuation approach: effects of an increase in the world interest rate when higher r raises steady-state firm size.

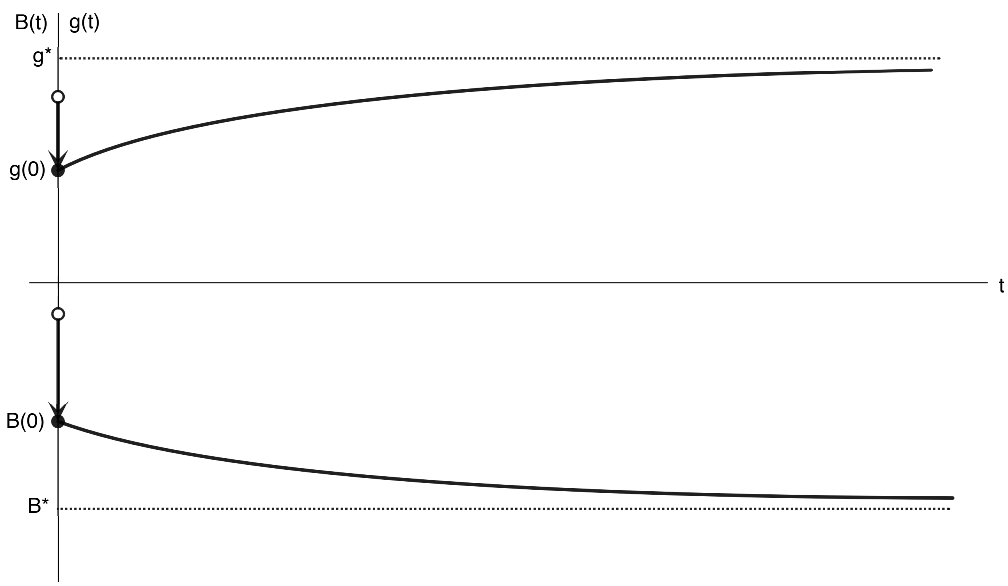

Figure A8: Valuation approach: impulse responses of growth and the foreign asset position when higher r raises steady-state firm size.

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